Unit 1 Algebra

Day 1 - Review Factoring

<u>Objective</u> - You will be able to factor the sum and difference of cubes and factor by grouping.

Types of Factoring

- GCF (always look for first!)
- DOTS
- 2 Binomials (3 terms)
- Factoring by Grouping (4 terms)

Greatest Common Factor

When factoring, always look for the GCF!

1) $35x^2 - 15x$

$\boxed{5x(7x-3)}$

2) $x^3y^4z + xy^3z$

$\boxed{xy^3z(x^2y+1)}$

Factor Completely

1) $25x^2 - 100$

$25(x^2-4)$

$25(x+2)(x-2)$

2) $3x^2 + 12x - 36$

$3(x^2+4x-12)$

$\boxed{3(x+6)(x-2)}$

Factoring by Grouping

Method of factoring used when there are 4 terms.

1) Take the GCF of the first pair of terms.
2) Take the GCF of the second pair of terms.
3) Factor out the GCF from the resulting terms (this is the term in parentheses - should be the same for both steps 1 & 2).

Examples

1) $x^2 - xy + 2x - 2y$

$x(x-y) + 2(x-y)$

$\boxed{(x+2)(x-y)}$

2) $x^2yz - xyz^2 + 2x - 2z$

$xyz(x-z) + 2(x-z)$

$\boxed{(xyz+2)(x-z)}$

3) $2a^3 - a^2b - 2a + b$

$a^2(2a-b) - 1(2a-b)$

$(a^2-1)(2a-b)$

$\boxed{(a+1)(a+1)(2a-b)}$

4) $\dfrac{ux + vx - uy - vy}{2ux + 2vx + uy + vy}$

$= \dfrac{x(u+v) - y(u+v)}{2x(u+v) + y(u+v)}$

$= \dfrac{(x-y)(u+v)}{(2x+y)(u+v)}$

$= \boxed{\dfrac{x-y}{2x+y}}$

Factoring Trinomials

1) $2x^2 - 7x - 15$

$2x^2 - 10x + 3x - 15$ $\quad M = -30$, $A = -7$, $-10, +3$

$2x(x-5) + 3(x-5)$

$\boxed{(2x+3)(x-5)}$

2) $6x^2 + 5x - 4$ $\quad M = -24$, $A = 5$

$6x^2 - 3x + 8x - 4$ $\quad -3, +8$

$3x(2x-1) + 4(2x-1)$

$\boxed{(3x+4)(2x-1)}$

Sum and Difference of Cubes

1) $m^3 - 125$

$= m^3 - 5^3$

$= (m-5)(m^2 - 5m + 25)$

2) $8x^3 + 27y^3$

$= (2x)^3 + (3y)^3$

$= (2x+3y)(2x^2 - 6xy + 9)$

General Rules

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Examples

1) $x^3 - 64$

$= (x)^3 - (4)^3$

$= (x-4)(x^2 + 4x + 16)$

2) $3x^3 - 81$

$3(x^3 - 27)$

$3((x)^3 - (3)^3)$

$= 3(x-3)(x^2 + 3x + 9)$

3) $-16x^3 - 2y^6$

$= -2(8x^3 + y^6)$

$= -2((2x)^3 + (y^2)^3)$

$= -2(2x + y^2)$

$(2x)^2 - 2xy^2 + y^4$

$= \boxed{-2(2x+y^2)(4x^2 - 2xy^2 + y^4)}$

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Day 2 - Complex Fractions (Day #1)

Objective - You will be able to reduce a complex fraction.

Complex Fractions

1) Determine the LCD of all terms
2) Multiply all terms by the LCD.
3) Combine Like terms if possible.
4) Factor if possible
5) Simplify if possible.

Examples

1) $$\frac{\frac{2}{3x}}{\frac{1}{x}}$$

$$= \frac{2}{3\cancel{x}} \cdot \frac{\cancel{x}}{1}$$

$$= \boxed{\frac{2}{3}}$$

2) $$\frac{\frac{2x^2}{y^2}}{\frac{4x}{y}}$$

$$\frac{\cancel{2}x^{\cancel{2}}}{y^{\cancel{2}}} \cdot \frac{\cancel{y}}{{}_{2}\cancel{4}\cancel{x}}$$

$$= \boxed{\frac{x}{2y}}$$

3) $$\frac{x\left(x - \frac{1}{x}\right)}{x\left(x+1\right)}$$

$$\frac{x^2 - 1}{x^2 + 1x} = \frac{\cancel{(x+1)}(x-1)}{x\cancel{(x+1)}}$$

$$= \boxed{\frac{x-1}{x}}$$

4) $$\frac{x^2(x+2)}{x^2\left(1 + \frac{5}{x} + \frac{6}{x^2}\right)}$$

$$= \frac{x^3 + 2x^2}{x^2 + 5x + 6}$$

$$= \frac{x^2\cancel{(x+2)}}{(x+3)\cancel{(x+2)}} = \boxed{\frac{x^2}{x+3}}$$

5) $$\frac{\left(\dfrac{y}{x+y} - \dfrac{x}{x-y}\right)(x+y)(x-y)}{(x+y)(x-y)\left(\dfrac{x}{x+y} + \dfrac{y}{x-y}\right)(x+y)(x-y)}$$

$$\frac{y(x-y) - x(x+y)}{x(x-y) + y(x+y)}$$

$$= \frac{\cancel{xy} - y^2 + x^2 - \cancel{xy}}{x^2 - xy + xy + y^2}$$

$$= \frac{-y^2 + x^2}{x^2 + y^2} = \boxed{-1}$$

Homework

WS #1-5.

Day 3 – Complex Fractions (con't)

Examples

1) $$\frac{x^2\left(1-\frac{2}{x}-\frac{24}{x^2}\right)x^2}{x^2\left(1-\frac{6}{x}\right)x^2}$$

$$=\frac{x^2-2x-24}{x^2-6x}$$

$$=\frac{\cancel{(x-6)}(x+4)}{x\cancel{(x-6)}}$$

$$=\boxed{\frac{x+4}{x}}$$

2) $$\frac{\left(x-\frac{1}{x}\right)\left(1-\frac{1}{x}\right)}{\left(\frac{1}{x}-\frac{1}{x^2}\right)\left(\frac{1}{x^3}-\frac{1}{x^2}\right)}$$

$$=\frac{x-1-\frac{1}{x}+\frac{1}{x^2}}{\frac{1}{x^4}-\frac{1}{x^3}-\frac{1}{x^5}+\frac{1}{x^6}}\cdot\frac{x^5}{x^5}$$

$$=\frac{x^6-x^5-x^4+x^3}{x-x^2-1+x}$$

$$=\frac{x^3(x^3-x^2-x+1)}{-x^2+2x-1}$$

$$=\frac{x^3\left(x^2(x-1)-1(x-1)\right)}{-x^2+2x-1}$$

$$=\frac{x^3(x^2-1)(x-1)}{-x^2+2x-1}$$

$$=\frac{x^3(x+1)(x-1)(x-1)}{-1(x^2-2x+1)}$$

$$=\frac{x^3(x+1)\cancel{(x-1)}\cancel{(x-1)}}{-1\cancel{(x-1)}\cancel{(x-1)}}$$

$$\boxed{[illegible](x+1)}$$

3) $$1-\frac{1}{1-\frac{1}{1-\frac{1}{1-x}}}$$

$$=1-\frac{1}{1-\frac{1}{\frac{1-x-1}{1-x}}}$$

$$=1-\frac{1}{1-\frac{1-x}{-x}}=1-\frac{1}{\frac{-x-1+x}{-x}}$$

$$=1-\frac{-x}{-1}=\boxed{1-x}$$

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Day #4 Rational Equations

Objective: You will be able to solve rational equations.

Rational Equations

Solving Equations involving fractions

1) Multiply all the terms by the common denominator.
2) Solve the resulting equation.

Note: You must check your solutions! To do this just make sure that the solutions do not make any of the fractions undefined.

Example #1

$$r(r-4)\left(\frac{r+4}{r} + \frac{3}{r-4} = \frac{-16}{r^2-4r}\right)r^2-4r$$

$$(r-4)(r+4) + 3r = -16$$

$$r^2 \cancel{+4r} - 4r - 16 + 3r + 16 = 0$$

$$r^2 - 3r = 0$$

$$r(r-3) = 0$$

$\boxed{r=0}$ reject | $r-3=0$, $\boxed{r=3}$

Check: $\frac{3+4}{3} + \frac{3}{3-4} = \frac{-16}{(3)^2-4(3)}$

$$\frac{7}{3} + \frac{3}{-1} = \frac{-16}{9-12}$$

$$-\frac{2}{3} =$$

Example #2

$$a(a-1)(a+1)(a-4)\left[a + \frac{a^2-5}{a^2-1} = \frac{-16}{a^2-4a}\right]$$

$a(a-4)(a-1)(a+1)$

$(a-1)(a+1)(a)(a-4)$

$$a(a^2-1)(a^2-4a) + a^2-5(a^2-4a) = -16(a^2-1)$$

$$(a^3-a)(a^2-4a) + a^4-4a^3-5a^2+20a = -16a^2+16$$

$$a^5\underbrace{-4a^4-a^3+4a^2+a^4}-4a^3-5a^2+20a = -16a^2+16$$

$$a^5-3a^4-5a^3-a^2+20a = -16a^2+16$$

$$+16a^2 \quad -16 \quad +16a^2 \quad -16$$

$$a^5-3a^4-5a^3+15a^2+20a-16=0$$

$$a^4(a-3)-5a^2(a-3)+4(5a-4)=0$$

$$(a^4-5a^2+4)(a-3)(5a-4)=0$$

$$(a^2-1)(a^2-4)(a-3)(5a-4)=0$$

$(a+1)$	$(a-1)$	$(a+2)$	$(a-2)$	$(a+3)$	$(5a-4)=0$
$\boxed{a=-1}$	$\boxed{a=1}$	$\boxed{a=-2}$	$\boxed{a=2}$	$\boxed{a=3}$	$\frac{5a}{5}=\frac{4}{5}$
reject	reject				$\boxed{a=4/5}$

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Day #5 Partial Fractions

Objectives: You will Decompose a fraction into partial fractions.

Partial Fractions

Decomposing a single fraction into the sum of two fractions.

1) Factor the denominator of the given fraction and seperate into the sum of two fractions.

2) Let "A" be the numerator of one fraction and "B" be the numerator of the other.

3) Set the sum equal to the original and multiply the terms by the LCD to eliminate the fractions.

4) To solve for "A", let X equal the number that would eliminate "B" and to solve for "B," let X equal the number that would eliminate "A".

5) Substitute "A" and "B" into the sum of two fractions.

Example #1

Decompose into partial fractions:

$$\frac{8y+7}{y^2-y-2}$$

$$\frac{8y+7}{(y-2)(y+1)} = \frac{A}{y-2} + \frac{B}{y+1}$$

So $\dfrac{\frac{1}{3}}{y-2} + \dfrac{7\,^2/_3}{y+1}$

$8y+7 = A(y+1) + B(y-2)$

$8(-1)+7 = A(-1+1) + B(-1-2)$

$\frac{-1}{-3} = \frac{-3b}{-3}$ $\boxed{B = 1/3}$

$8(2)+7 = A(2+1) + B(2-2)$

$\frac{23}{3} = \frac{3a}{3}$ $\boxed{A = 7\,^2/_3}$

Example #2

Decompose into partial fractions:

$$\frac{2x}{2x^2+5x-3}$$

$$= \frac{2x}{(2x-1)(x+3)} = \frac{A}{2x-1} + \frac{B}{x+3}$$

$$2x = A(x+3) + B(2x-1)$$

$$2(1/2) = A(1/2+3) + B(2(1/2)-1)$$

$$\frac{1}{3.5} = \frac{3.5A}{3.5}$$

$$\boxed{A = \frac{2}{7}}$$

$$2(-3) = A(-3+3) + B(2(-3)-1)$$

$$\frac{-6}{-7} = \frac{-7B}{-7}$$

$$\boxed{B = \frac{6}{7}}$$

$$\frac{\frac{2}{7}}{2x-1} + \frac{\frac{6}{7}}{x+3}$$

Homework

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Day 6 - Systems with Three Variables

Objective - You will solve a system with 3 equations + 3 unknowns algebraically.

Solving Systems of Equations

Solve the following systems algebraically:

1) $3(3x-2y=8)$
$-2(5x-3y=16)$

$=\ 9x-6y=24$
$-10x+6y=-32$

$-1x=-8$
$\frac{-1x}{-1}=\frac{-8}{-1}$
$\boxed{x=8}$

$3(8)-2y=8$
$24-2y=8$
$\frac{-2y}{-2}=\frac{-16}{-2}$
$\boxed{y=8}$

$(8,8)$

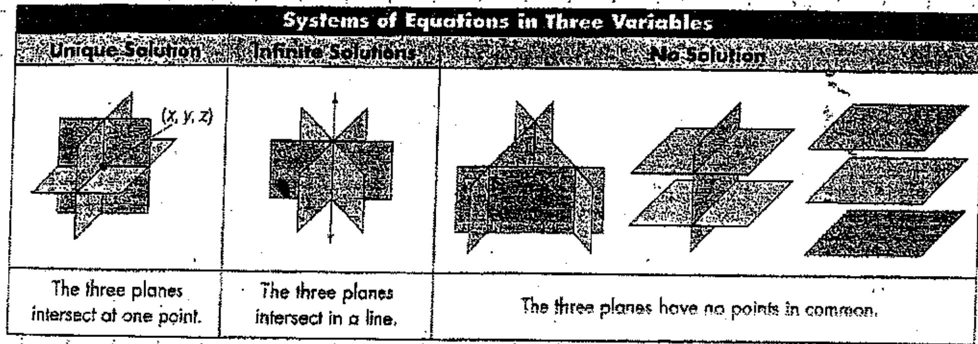
2) $-6(2x+3y=3) \rightarrow$
$12x-15y=-4$

$-12x-18y=-18$
$+12x-15y=-4$

$\frac{-33y}{-33}=\frac{-22}{-33}$
$\boxed{y=\frac{2}{3}}$

$2x+3(2/3)=3$
$2x+2=3$
$-2\quad -2$
$\frac{2x}{2}=\frac{1}{2}$
$\boxed{x=1/2}$

$(1/2,\ 2/3)$



Solving a 3 Variable System.

1) Select a pair of equations and eliminate one variable, leaving one equation with 2 variables.

2) Repeat step 1 using a different pair of equations and eliminating the same variable.

3) Solve the resulting two equations with two variables

4) Work backwards to find the remaining variables.

Example #1

Solve algebraically
① $3x - 5y + z = 9$
② $x - 3y - 2z = -8$
③ $5x - 6y + 3z = 15$

① $3x - 5y + z = 9$
② $(x - 3y - 2z = -8) \cdot -3$

① $3x - 5y + z = 9$
④ $-3x + 9y + 6z = +24$
⑤ $4y + 7z = 33$

② $(x - 3y - 2z = -8) \cdot -5$
③ $5x - 6y + 3z = 15$

⑥ $-\cancel{5x} + 15y + 10z = 40$
③ $\cancel{5x} - 6y + 3z = 15$
⑦ $9y + 13z = 55$

⑤ $(4y + 7z = 33) \cdot -9$
⑦ $(9y + 13z = 55) \cdot 4$

$-\cancel{36y} - 63z = -297$
$\cancel{36y} + 52z = 220$
$-11z = -77$
$\frac{-11z}{-11} = \frac{-77}{-11}$
$\boxed{z = 7}$

⑤ $4y + 7(7) = 33$
$4y + 49 = 33$
$4y = -16$ $\boxed{y = -4}$

② $x - 3(-4) - 2(7) = -8$
$x + 12 - 14 = -8$
$\boxed{x = -6}$
$\boxed{(-6, -4, 7)}$

Example #2

Solve Algebraically:
$$\begin{aligned} ①\ & 4x + 2y + z = 7 \\ ②\ & 2x + 2y - 4z = -4 \\ ③\ & x + 3y - 2z = -8 \end{aligned}$$

$$\begin{aligned} ①\ & 4x + 2y + z = 7 \\ ②\ & -2(2x + 2y - 4z = -4) \\ \hline ①\ & \cancel{4x} + 2y + z = 7 \\ ④\ & \cancel{-4x} - 4y + 8z = 8 \\ \hline \end{aligned}$$

$$⑤\ -2y + 9z = 15$$

$$\begin{aligned} ②\ & 2x + 2y - 4z = -4 \\ ③\ & (x + 3y - 2z = -8)\,-2 \\ \hline ②\ & \cancel{2x} + 2y - \cancel{4z} = -4 \\ & \cancel{-2x} - 6y + \cancel{4z} = 16 \\ \hline \end{aligned}$$

$$\frac{-4y}{-4} = \frac{12}{-4}$$

$$\boxed{y = -3}$$

$$⑤\ -2(-3) + 9z = 15$$

$$\begin{aligned} 6 + 9z &= 15 \\ -6 \quad & \quad -6 \end{aligned}$$

$$\frac{9z}{9} = \frac{9}{9}$$

$$\boxed{z = 1}$$

$$①\ 4x + 2(-3) + 1 = 7$$

$$4x - 6 + 1 = 7$$

$$4x - 5 = 7$$

$$\frac{4x}{4} = \frac{12}{4}$$

$$\boxed{x = 3}$$

$$\boxed{(3, -3, 1)}$$

Homework

Solve the following system of equations algebraically:

$$\begin{aligned} x + 2y + 3z &= 5 \\ 3x + 2y - 2z &= -13 \\ 5x + 3y - z &= -11 \end{aligned}$$

Day 7 - Systems of Linear Inequalities

Objective - you will graph a system of inequalities and find the min or max value of a function defined by the convex polygonal set.

Solving Systems of Linear Inequalities

Polygonal Convex Set

a bounded set of all points on or inside a convex polygon graphed on the coordinate plane.

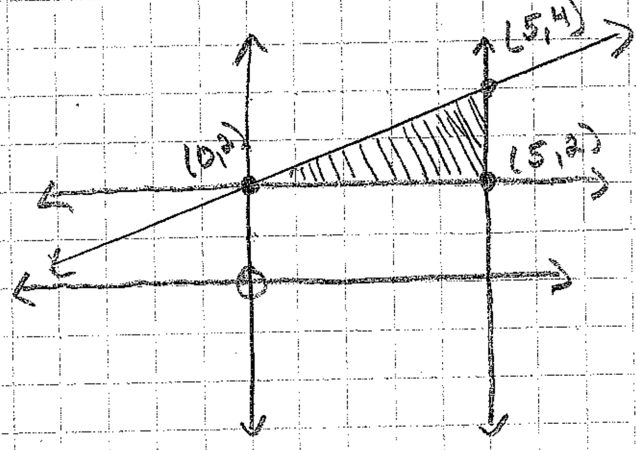
Vertex Theorem

to find the max/min of a function, only the verticies need be evaluated in the function.

Example #1

Find the maximum and minimum of the following function:

$$f(x,y) = 3x + y$$
$$x \le 5$$
$$y \ge 2$$
$$2x - 5y \ge -10 \Rightarrow$$



$$-5y \ge -2x - 10$$
$$\frac{-5y}{-5} \quad \frac{-2x}{-5} \quad \frac{-10}{-5}$$
$$y \ge \frac{2}{5}x + 2$$

$f(x,y) = 3x + y$

$f(0,2) = 3(0) + 2 = 2$

$f(5,2) = 3(5) + 2 = 17$

$f(5,4) = 3(5) + 4 = 19$

$\therefore$ $\boxed{\begin{array}{l} \text{Min} = 2 \\ \text{Max} = 19 \end{array}}$

Example #2: Find the maximum and minimum of the following function:

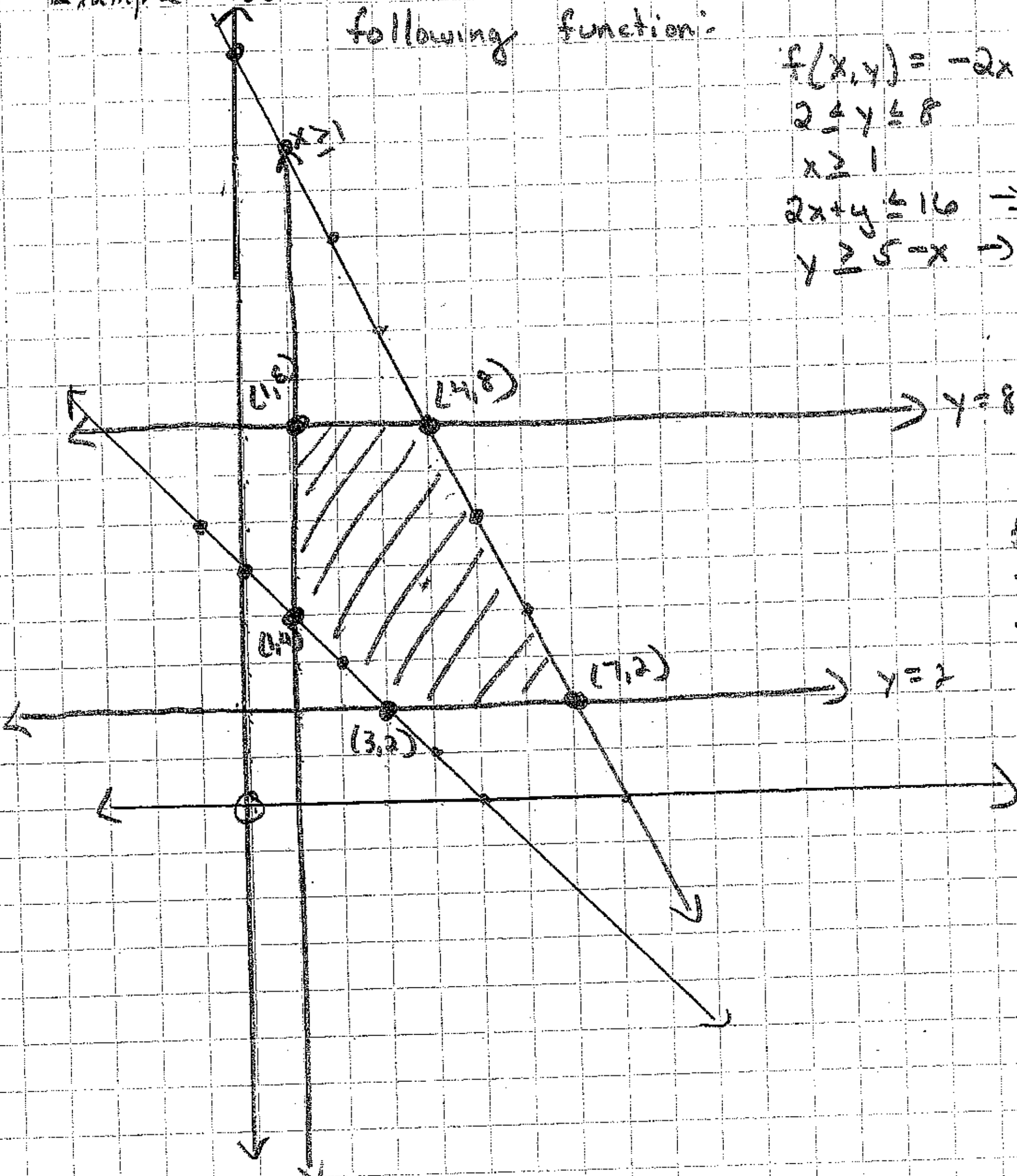
$f(x,y) = -2x + y + 5$

$2 \le y \le 8$

$x \ge 1$

$2x + y \le 16 \rightarrow y \le -2x + 16$

$y \ge 5 - x \rightarrow y \ge -x + 5$



$f(x,y) = -2x + y + 5$

$f(1,8) = -2(1) + 8 + 5 = 11$

$f(4,8) = -2(4) + 8 + 5 = 5$

$f(7,2) = -2(7) + 2 + 5 = -7$

$f(3,2) = -2(3) + 2 + 5 = 1$

$f(1,4) = -2(1) + 4 + 5 = 7$

$\therefore$ $\boxed{\text{Max} = 11 \\ \text{Min} = -7}$

Homework: Find the maximum and minimum of the following function:

$f(x,y) = x - y + 2$

$x + 4y \le 12$

$3x - 2y \ge -6$

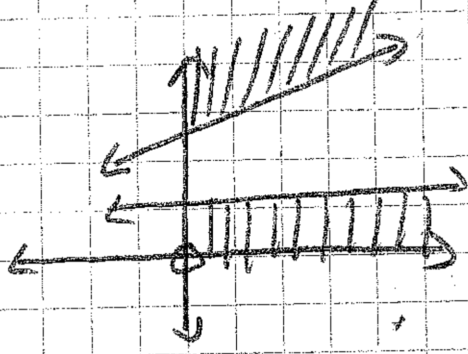
$x + y \ge -2$

$3x - y \le 10$

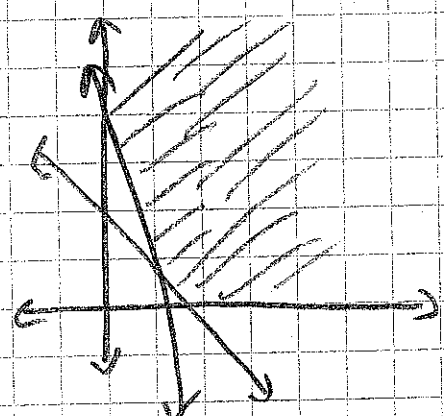
Day 8- Linear Programming

Objective- You will solve applications of a system of inequalities.

Linear Programming



Infeasible - when max/min cannot be determined.



Unbounded - you can find a minimum value but not a maximum value.

Procedure

1) Define the variables

2) Write the constraints as a system of inequalities.

3) Write the expression whose value is to be min/max as a function of two variables.

4) Graph the system found in step 2 and find the coordinates of the verticies of the polygon.

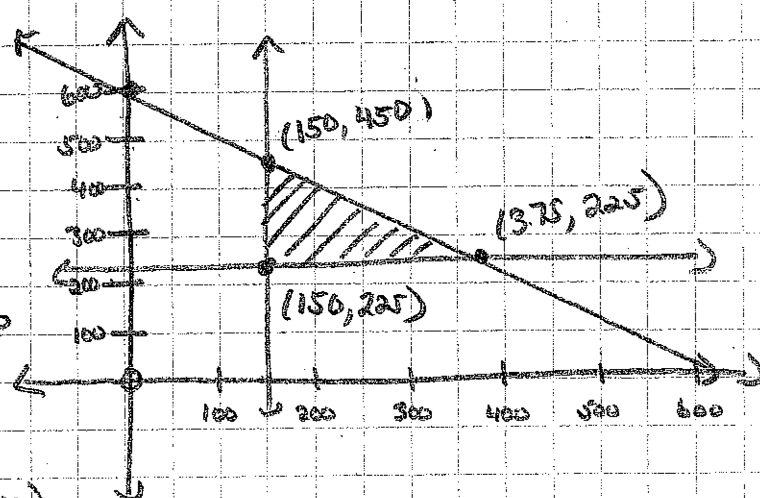
5) Substitute the verticies into the function found in step 3.

6) Select the max or min values.

Example

Suppose a lumber mill can turn out 600 units of product each week. To meet the needs of its regular customers, the mill must produce 150 units of lumber and 225 units of plywood. If the profit for each unit of lumber is $30 and the profit for each unit of plywood is $45, how many units of each type of wood product should the mill produce to maximize profit?

Let x = lumber
Let y = plywood.



$x \geq 150$
$y \geq 225$
$x + y \leq 600 \rightarrow y \leq -x + 600$

$f(x,y) = 30x + 45y$

$f(150,450) = 30(150) + 45(450)$
$= \$24,750 \leftarrow$

$f(375,225) = 30(375) + 45(225)$
$= \$21,375$

$f(150,225) = 30(150) + 45(225)$
$= 14,625$

$\therefore$ 150 Lumber
450 Plywood.

Homework
WS #1